MTH 112 Spring 2021

Quiz 9 **SOLUTIONS**

Solve the following problems.

1.) Determine whether the following series converge or diverge. If a series converges, what does it converge to?

a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

Solution: Since the ratio $r = 2/3$ is in the range $-1 < r < 1$, this geometric series converges. Notice that the first term of the series is $2/3$.

$$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n = \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 + \cdots$$
$$= \frac{2/3}{1-2/3} = \frac{2}{3}\cdot\frac{3}{1} = 2 \quad \blacksquare$$

b) $\displaystyle\sum_{n=1}^{\infty}\left(-\frac{5}{4}\right)^n$

Solution: Since the ratio $r = -5/3$ is less than or equal to -1, this geometric series diverges. $\blacksquare$

2.) Determine whether the following series converge or diverge. You must explain your answer by stating the appropriate test used and showing any work needed to justify your conclusion.

a) $\displaystyle\sum_{n=1}^{\infty}\frac{2n^2-3}{n^2}$

Solution: Since

$$\lim_{n\to\infty}\frac{2n^2-3}{n^2} = \lim_{n\to\infty}\left(2-\frac{3}{n^2}\right) = 2 \neq 0,$$

this series diverges by the Divergence Test (or the Nth Term Test). $\blacksquare$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2}{n^3+1}$

Solution: Since the terms are given by a positive, decreasing, and continuous function, we can apply the Integral Test.

$$\begin{aligned}\int_2^{\infty} \frac{x^2}{x^3+1}\,dx &= \lim_{R\to\infty} \int_2^{R} \frac{x^2}{x^3+1}\,dx \\ &= \lim_{R\to\infty} \frac{1}{3}\ln(x^3+1)\Big|_2^R \\ &= \lim_{R\to\infty} \frac{1}{3}\ln(R^3+1) - \frac{1}{3}\ln(9) = \infty\end{aligned}$$

So, the series above diverges by the Integral Test. ■