MTH 112 Spring 2021

Quiz 8 **Solutions**

Solve the following problems. Be sure to write any improper integral as the correct limit statement.

1.) $\displaystyle\int_0^2 \frac{1}{\sqrt{2-x}}\,dx$

Solution:

$$\begin{aligned}\int_0^2 \frac{1}{\sqrt{2-x}}\,dx &= \lim_{r\to 2^-}\int_0^r \frac{1}{\sqrt{2-x}}\,dx \\ &= \lim_{r\to 2^-} -2\sqrt{2-x}\Big|_0^r \\ &= \lim_{r\to 2^-} -2\sqrt{2-r}+2\sqrt{2} = 2\sqrt{2}\ \blacksquare\end{aligned}$$

2.) A r.v. Y has sample space $S=[2,\infty)$ and pdf of the form $f(y)=\dfrac{A}{y^3}$.

a) Find the value of A that normalizes the pdf.

Solution:

$$\begin{aligned}1 &= \int_2^\infty \frac{A}{y^3}\,dy = \lim_{R\to\infty}\int_2^R \frac{A}{y^3}\,dy \\ &= \lim_{R\to\infty} A\left[-\frac{1}{2y^2}\right]_2^R = A\lim_{R\to\infty}\left[-\frac{1}{2R^2}+\frac{1}{8}\right] \\ &= \frac{A}{8}\end{aligned}$$

So, $A=8$. $\blacksquare$

b) What is the probability that an observation for Y is in the range $2\le Y\le 4$?

Solution:

$$\begin{aligned}\mathbb{P}(2\le Y\le 4) &= \int_2^4 \frac{8}{y^2}\,dy \\ &= \left[-\frac{4}{y^2}\right]_2^4 \\ &= -\frac{1}{4}+1 = \frac{3}{4}\ \blacksquare\end{aligned}$$