MTH 112 Spring 2021

Quiz 7 **Solutions**

Compute the following indefinite integrals.

1.) $\displaystyle\int \frac{2x^3+5x^2-5x+6}{x^2+2x-3}\,dx$

**Solution:** We first perform long division to get

$$\frac{2x^3+5x^2-5x+6}{x^2+2x-3} = 2x+1+\frac{-x+9}{x^2+2x-3}.$$

The correct partial fraction decomposition for the proper rational function is

$$\frac{-x+9}{x^2+2x-3} = \frac{-x+9}{(x+3)(x-1)} = \frac{A}{x+3}+\frac{B}{x-1}.$$

The equation we get for the unknown coefficients is

$$-x+9 = A(x-1)+B(x+3).$$

Substituting $x=-3$ and $x=1$ gives us $A=-3$ and $B=2$. So, the original integral is computed as follows.

$$\begin{aligned}\int \frac{2x^3+5x^2-5x+6}{x^2+2x-3}\,dx &= \int 2x+1-\frac{3}{x+3}+\frac{2}{x-1}\,dx\\ &= x^2+x-3\ln|x+3|+2\ln|x-1|+C. \blacksquare\end{aligned}$$

2.) $\displaystyle\int \frac{2x^2-x+32}{x^3+16x}\,dx$

**Solution:** The correct partial fraction decomposition for this proper rational function is

$$\frac{2x^2-x+32}{x^3+16x} = \frac{2x^2-x+32}{x(x^2+16)} = \frac{A}{x}+\frac{Bx+C}{x^2+16}.$$

That means we need

$$\begin{aligned}2x^2-x+32 &= A(x^2+16)+(Bx+C)x\\ &= (A+B)x^2+Cx+16A.\end{aligned}$$

Matching coefficients, we need $A=2, B=0$, and $C=-1$. This gives us the following integral.

$$\begin{aligned}\int \frac{2x^2-x+32}{x^3+16x}\,dx &= \int \frac{2}{x}\,dx - \int \frac{1}{x^2+16}\,dx\\ &= 2\ln|x| - \frac{1}{4}\int \frac{1}{\left(\frac{x}{4}\right)^2+1}\,\frac{1}{4}dx\\ &= 2\ln|x| - \frac{1}{4}\arctan\left(\frac{x}{4}\right)+C. \blacksquare\end{aligned}$$