

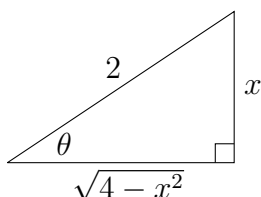
MTH 112 Spring 2021

Quiz 6 **Solutions**

Compute the following indefinite integrals.

1.) $\int \frac{1}{(4-x^2)^{3/2}} dx$

Solution: The following triangle gives us the appropriate substitution.



$$\begin{aligned}\frac{x}{2} &= \sin(\theta) \\ x &= 2 \sin(\theta) \\ dx &= 2 \cos(\theta) d\theta\end{aligned}$$

We also see that

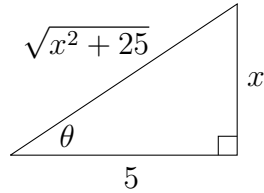
$$\begin{aligned}\frac{\sqrt{4-x^2}}{2} &= \cos(\theta), \\ \sqrt{4-x^2} &= 2 \cos(\theta).\end{aligned}$$

We can now evaluate the integral.

$$\begin{aligned}\int \frac{1}{(4-x^2)^{3/2}} dx &= \int \frac{1}{8 \cos^3(\theta)} \cdot 2 \cos(\theta) d\theta \\ &= \frac{1}{4} \int \sec^2(\theta) d\theta \\ &= \frac{1}{4} \tan(\theta) + C \\ &= \frac{x}{4\sqrt{4-x^2}} + C \blacksquare\end{aligned}$$

$$2.) \int \frac{1}{\sqrt{x^2 + 25}} dx$$

Solution: The following triangle gives us the appropriate substitution.



$$\begin{aligned}\frac{x}{5} &= \tan(\theta) \\ x &= 5 \tan(\theta) \\ dx &= 5 \sec^2(\theta) d\theta\end{aligned}$$

We also see that

$$\begin{aligned}\frac{5}{\sqrt{x^2 + 25}} &= \cos(\theta), \\ \frac{1}{\sqrt{x^2 + 25}} &= \frac{\cos(\theta)}{5}.\end{aligned}$$

We can now evaluate the integral.

$$\begin{aligned}\int \frac{1}{\sqrt{x^2 + 25}} dx &= \int \frac{\cos(\theta)}{5} \cdot 5 \sec^2(\theta) d\theta \\ &= \int \sec(\theta) d\theta \\ &= \ln |\sec(\theta) + \tan(\theta)| + C \\ &= \ln \left| \frac{\sqrt{x^2 + 25}}{5} + \frac{x}{5} \right| + C \blacksquare\end{aligned}$$