MTH 112 Spring 2021

Quiz 5 **Solutions**

Compute the following indefinite integrals.

1.) $\displaystyle\int x^3 \ln(x)\, dx$

Solution: Choosing

$$\begin{aligned} u &= \ln(x), \quad & dv &= x^3\, dx, \\ du &= \tfrac{1}{x}\, dx, & v &= \tfrac{1}{4}x^4, \end{aligned}$$

we find

$$\begin{aligned} \int x^3 \ln(x)\, dx &= \frac{1}{4}x^4 \ln(x) - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\, dx \\ &= \frac{1}{4}x^4 \ln(x) - \frac{1}{4}\int x^3\, dx \\ &= \frac{1}{4}x^4 \ln(x) - \frac{1}{16}x^4 + C. \ \blacksquare \end{aligned}$$

2.) $\displaystyle\int x^2 e^{3x}\, dx$

Solution: Choosing

$$\begin{aligned} u &= x^2, & dv &= e^{3x}\, dx, \\ du &= 2x\, dx, & v &= \tfrac{1}{3}e^{3x}, \end{aligned}$$

we find

$$\begin{aligned} \int x^2 e^{3x}\, dx &= \frac{1}{3}x^2 e^{3x} - \int \frac{1}{3}e^{3x}(2x)\, dx \\ &= \frac{1}{3}x^2 e^{3x} - \frac{2}{3}\int x e^{3x}\, dx. \end{aligned}$$

Integrating by parts again with

$$\begin{aligned} u &= x, & dv &= e^{3x}\, dx, \\ du &= dx, & v &= \tfrac{1}{3}e^{3x}, \end{aligned}$$

gives

$$\begin{aligned} \int x^2 e^{3x}\, dx &= \frac{1}{3}x^2 e^{3x} - \frac{2}{3}\left[\frac{1}{3}x e^{3x} - \int \frac{1}{3}e^{3x}\, dx\right] \\ &= \frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{9}\int e^{3x}\, dx \\ &= \frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + C. \ \blacksquare \end{aligned}$$

3.) $\displaystyle\int \sin^4(\theta)\cos^5(\theta)\ d\theta$

Solution: We pair one of the factors of cosine with $d\theta$ and make the substitution $u = \sin(\theta)$.

$$\begin{aligned}
\int \sin^4(\theta)\cos^5(\theta)\ d\theta &= \int \sin^4(\theta)\left(\cos^2(\theta)\right)^2\ \cos(\theta)\ d\theta \\
&= \int \sin^4(\theta)\left(1-\sin^2(\theta)\right)^2\ \cos(\theta)\ d\theta \\
&= \int u^4(1-u^2)^2\ du \\
&= \int u^4(1-2u^2+u^4)\ du \\
&= \int (u^4-2u^6+u^8)\ du \\
&= \frac{1}{5}u^5 - \frac{2}{7}u^7 + \frac{1}{9}u^9 + C \\
&= \frac{1}{5}\sin^5(\theta) - \frac{2}{7}\sin^7(\theta) + \frac{1}{9}\sin^9(\theta) + C \ \blacksquare
\end{aligned}$$

4.) $\displaystyle\int \tan^2(\theta)\sec^4(\theta)\ d\theta$

Solution: We pair $\sec^2(\theta)$ with $d\theta$ and make the substitution $u = \tan(\theta)$.

$$\begin{aligned}
\int \tan^2(\theta)\sec^4(\theta)\ d\theta &= \int \tan^2(\theta)\sec^2(\theta)\ \sec^2(\theta)\ d\theta \\
&= \int \tan^2(\theta)(\tan^2(\theta)+1)\ \sec^2(\theta)\ d\theta \\
&= \int u^2(u^2+1)\ du \\
&= \int (u^4+u^2)\ du \\
&= \frac{1}{5}u^5 + \frac{1}{3}u^3 + C \\
&= \frac{1}{5}\tan^5(\theta) + \frac{1}{3}\tan^3(\theta) + C \ \blacksquare
\end{aligned}$$