MTH 112 Spring 2021

Quiz 3 **Solutions**

1.) Find the arc length of the curve $y = \dfrac{x^4}{16} + \dfrac{1}{2x^2}$ on the interval $1 \leq x \leq 2$.

Solution:

$$\begin{aligned}
L &= \int_1^2 \sqrt{1 + [f'(x)]^2}\, dx \\
&= \int_1^2 \sqrt{1 + \left[\frac{x^3}{4} - \frac{1}{x^3}\right]^2}\, dx \\
&= \int_1^2 \sqrt{1 + \frac{x^6}{16} - \frac{1}{2} + \frac{1}{x^6}}\, dx \\
&= \int_1^2 \sqrt{\frac{x^6}{16} + \frac{1}{2} + \frac{1}{x^6}}\, dx \\
&= \int_1^2 \sqrt{\left[\frac{x^3}{4} + \frac{1}{x^3}\right]^2}\, dx \\
&= \int_1^2 \frac{x^3}{4} + \frac{1}{x^3}\, dx \\
&= \frac{x^4}{16} - \frac{1}{2x^2}\Big|_1^2 \\
&= \left(\frac{16}{16} - \frac{1}{8}\right) - \left(\frac{1}{16} - \frac{1}{2}\right) \\
&= \frac{21}{16} \quad \blacksquare
\end{aligned}$$

2.) Find the surface area of the solid generated by rotating the curve $y = 2x^3$ on $0 \leq x \leq 1$ about the x-axis.

Solution:

$$\begin{aligned} SA &= \int_0^1 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx \\ &= \int_0^1 2\pi(2x^3)\sqrt{1+(6x^2)^2}\,dx \\ &= 4\pi\int_0^1 x^3\sqrt{1+36x^4}\,dx \end{aligned}$$

Let $u = 36x^4 + 1$ and so $du = 144x^3\,dx$.

$$\begin{aligned} SA &= \frac{4\pi}{144}\int_0^1 144x^3\sqrt{1+36x^4}\,dx \\ &= \frac{\pi}{36}\int_1^{37}\sqrt{u}\,du \\ &= \frac{\pi}{36}\left[\frac{2}{3}u^{3/2}\right]_1^{37} \\ &= \frac{\pi}{54}\left(37\sqrt{37}-1\right) \quad \blacksquare \end{aligned}$$