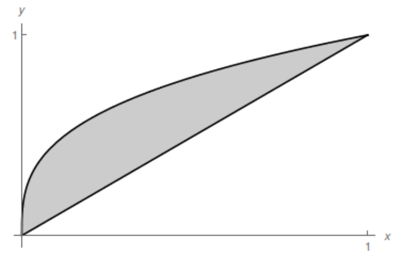
MTH 112 Spring 2021

Quiz 2 **SOLUTIONS**

1.) Find the volume of the solid generated by rotating the region contained between the curves $y = x$ and $y = x^{1/3}$ in the first quadrant around the x-axis.



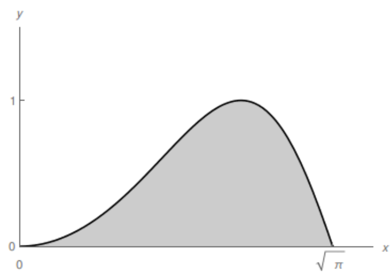
Solution: First, the curves intersect at

$$\begin{aligned} x &= x^{1/3} \\ x^3 &= x \\ x(x^2 - 1) &= 0 \\ x &= -1, 0, 1. \end{aligned}$$

So, the intersections in the first quadrant are at $x = 0$ and $x = 1$. Rotating about the x-axis gives a solid having washers as cross-sections. The outer radius is $y = x^{1/3}$ while the inner radius is $y = x$. So, the volume is

$$\begin{aligned} V &= \int_0^1 \pi \left[\left(x^{1/3}\right)^2 - x^2 \right] \, dx \\ &= \pi \int_0^1 x^{2/3} - x^2 \, dx \\ &= \pi \left(\frac{3}{5} x^{5/3} - \frac{1}{3} x^3 \Big|_0^1 \right) \\ &= \pi \left(\frac{3}{5} - \frac{1}{3} \right) = \frac{4\pi}{15}. \quad \blacksquare \end{aligned}$$

2.) Find the volume of the solid generated by rotating the area under $y = \sin(x^2)$ on $0 \le x \le \sqrt{\pi}$ around the y-axis. (HINT: Using washers would be a BIG mistake!)



Solution: The easiest method to compute this volume is using cylindrical shells. Since we are rotating around the y-axis, the radius of shell is simply x while the height of the shell is $y = \sin(x^2)$. This gives the volume as

$$\begin{aligned} V &= \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\, dx \\ &= 2\pi \int_0^{\sqrt{\pi}} x \sin(x^2)\, dx. \end{aligned}$$

Making the u-substitution

$$\begin{aligned} u &= x^2 \\ du &= 2x\, dx, \end{aligned}$$

$$\begin{aligned} V &= \pi \int_0^{\pi} \sin(u)\, du \\ &= \pi \left(-\cos(u)\big|_0^{\pi} \right) \\ &= \pi(-\cos(\pi) + \cos(0)) = 2\pi. \quad \blacksquare \end{aligned}$$