MTH 112 Sprin 2021

Quiz 1 **SOLUTIONS**

1.) Compute the following definite integral:

$$\int_0^1 \frac{x^3}{(1+x^4)^3}\,dx.$$

Solution: If we let $u = 1+x^4$, then $du = 4x^3\,dx$. Making this substitution (and changing the limits of integration) gives

$$\begin{aligned}
\int_0^1 \frac{x^3}{(1+x^4)^3}\,dx &= \frac{1}{4}\int_0^1 \frac{4x^3}{(1+x^4)^3}\,dx \\
&= \frac{1}{4}\int_1^2 \frac{1}{u^3}\,du \\
&= \frac{1}{4}\cdot \frac{u^{-2}}{-2}\Big|_1^2 \\
&= -\frac{1}{8}\left(\frac{1}{4}-1\right) \\
&= -\frac{1}{8}\left(-\frac{3}{4}\right) = \frac{3}{32}. \quad \blacksquare
\end{aligned}$$

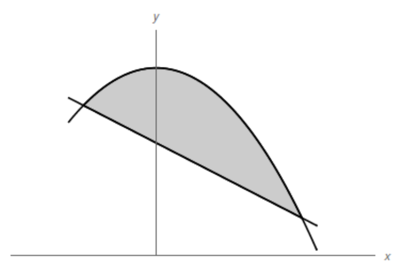
2.) Find the following indefinite integral:

$$\int \frac{\cos(2y)}{\sin(2y)+1}\,dy.$$

Solution: If we let $u = \sin(2y) + 1$, then $du = 2\cos(2y)\,dy$. This gives

$$\begin{aligned}\int \frac{\cos(2y)}{\sin(2y)+1}\,dy &= \frac{1}{2}\int \frac{2\cos(2y)}{\sin(2y)+1}\,dy \\ &= \frac{1}{2}\int \frac{1}{u}\,du \\ &= \frac{1}{2}\ln|u| + C \\ &= \frac{1}{2}\ln|\sin(2y)+1| + C. \quad \blacksquare\end{aligned}$$

3.) Compute the area of the region enclosed by the curves $y = 5 - x^2$ and $y = 3 - x$.



Solution: We need to find the points of intersection for these curves:

$$\begin{aligned}5 - x^2 &= 3 - x \\ x^2 - x - 2 &= 0 \\ (x+1)(x-2) &= 0 \\ x &= -1, 2.\end{aligned}$$

Since the parabola is clearly the upper curve, we can compute the area as follows:

$$\begin{aligned}
A &= \int_{-1}^{2} (5 - x^2) - (3 - x)\ dx \\
&= \int_{-1}^{2} 2 + x - x^2\ dx \\
&= 2x + \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_{-1}^{2} \\
&= \left(4 + 2 - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right) \\
&= 6 - \frac{8}{3} + 2 - \frac{1}{2} - \frac{1}{3} \\
&= 8 - \frac{1}{2} - \frac{9}{3} \\
&= 5 - \frac{1}{2} \\
&= \frac{9}{2}. \quad \blacksquare
\end{aligned}$$