

# EXAM 3 REVIEW PROBLEMS

1.) Compute the following integrals.

a)  $\int \frac{1}{(x^2 + 4)^2} dx$

b)  $\int \frac{x^2}{(9 - x^2)^{3/2}} dx$

2.) Answer the following:

a) Compute  $\int \frac{x^2 + 4x - 39}{x^2 + 3x - 18} dx$ .

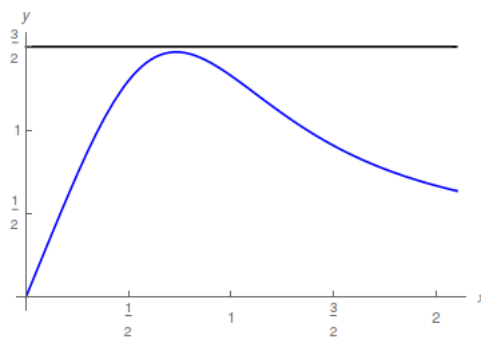
b) Compute  $\int \frac{3x^2 + x + 10}{x^3 + 2x^2 + 5x} dx$

c) Setup a partial fraction decomposition for the following rational function. You do not need to find the coefficients.

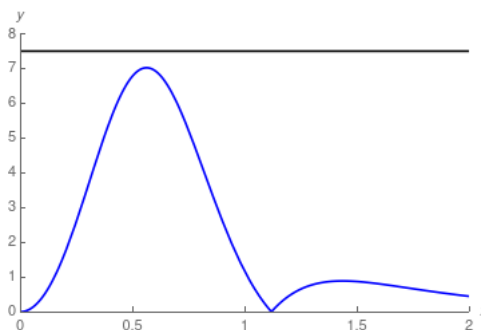
$$\frac{3x^2 + 15x - 2}{(x - 4)^3(x^2 + 4x + 9)}$$

2.) Suppose we want to approximate the integral  $\int_0^2 \sqrt{x^3 + 1} dx$  and we need to guarantee that the error in the approximation is no worse than  $1/100$ .

a) What is the minimal number of steps needed for the Trapezoid Rule? Below is a graph of the second derivative of  $\sqrt{x^3 + 1}$ .



b) What if we use Simpson's Rule instead? Below is a graph of the fourth derivative of the integrand.



3.) Evaluate the following improper integral (or show it does not exist). Be sure to write the given integral as the correct limit statement before solving!

$$\int_0^1 \frac{x+1}{\sqrt{x^2+2x}} dx$$

4.) Suppose that a random variable  $X$  has sample space  $S = [0, \infty)$  and a pdf of the form

$$f(x) = \frac{A}{x^2 + 1}.$$

- a) What coefficient  $A$  normalizes the pdf?
- b) What is the probability that an observation of  $X$  lies in the range  $[0, 1]$ ?
- c) What can you say about the expected value of this random variable?

5.) Suppose that a random variable  $P$  has sample space  $[0, 1]$  and pdf

$$f(p) = 60p^3(1-p)^2.$$

- a) What is the expected value of this random variable?
- b) What is the standard deviation of  $P$ ?