MTH 112 : Spring 2021
Formula Sheet for Exam 3

- Important Area Formulas
  - The area of a disk of radius r is given by $A = \pi r^2$.
  - The area of an annulus (or washer) with outer radius R and inner radius r is given by
$$A = \pi \left[R^2 - r^2\right].$$
  - The surface area of a cylindrical shell with radius r and height h is given by
$$A = 2\pi r h.$$
- Arc Length and Surface Area
  - The **arc length**, L, of a curve $y = f(x)$ on an interval $[a, b]$ is given by
$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx,$$
provided the derivative is continuous on (a, b).
  - The **surface area**, S, generated by revolving the graph of $y = f(x)$ (for $f(x) \geq 0$) on an interval $[a, b]$ about the x-axis is given by
$$S = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}\, dx,$$
provided the derivative is continuous on (a, b).
- Work
  - The total work done by a variable force $F = F(x)$ moving an abject from $x = a$ to $x = b$ is
$$W = \int_a^b F(x)dx.$$
  - The work done against gravity (near the surface of a planet) in moving an object with mass m vertically upward a distance h is $W = mgh$ where g is the acceleration due to gravity near the surface.
- Center of Mass
  - The center of mass for a planar figure is the point $(\overline{x}, \overline{y})$ where
$$\overline{x} = \frac{M_y}{M},$$
$$\overline{y} = \frac{M_x}{M}.$$

- In the formulas above,

$$M = \int_a^b dm,$$
$$M_y = \int_a^b \widetilde{x}\, dm,$$
$$M_x = \int_a^b \widetilde{y}\, dm,$$

  where dm is the mass of an (infinitesimal) strip and $(\widetilde{x}, \widetilde{y})$ is the center of the strip.

- Half Angle Formulas
  - $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$
  - $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
  - $\sinh^2(x) = \frac{1}{2}(\cosh(2x) - 1)$
  - $\cosh^2(x) = \frac{1}{2}(\cosh(2x) + 1)$

- The Trapezoid Rule

  The N-th Trapezoid Approximation to $\displaystyle\int_a^b f(x)\, dx$ is given by

$$T_N = \frac{\Delta x}{2}\Big(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{N-1} + y_N\Big).$$

  The error in this rule is approximated by

$$\left|\int_a^b f(x)\, dx - T_N\right| \leq \frac{M_2(b-a)^3}{12N^2}$$

  where M_2 is any upper bound on the second derivative:

$$0 \leq |f''(x)| \leq M_2 \ \text{ on } \ a \leq x \leq b$$

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- Simpson's Rule

  The N-th Approximation by Simpson's Rule (*N must be even*) to $\displaystyle\int_a^b f(x)\, dx$ is given by

$$S_N = \frac{\Delta x}{3}\Big(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{N-2} + 4y_{N-1} + y_N\Big).$$

  The error in this rule is approximated by

$$\left|\int_a^b f(x)\, dx - S_N\right| \leq \frac{M_4(b-a)^5}{180N^4}$$

  where M_4 is any upper bound on the fourth derivative:

$$0 \leq |f''''(x)| \leq M_4 \ \text{ on } \ a \leq x \leq b$$

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- Continuous Probability

  For a random variable X with sample space $S = [a, b]$ and pdf $f = f(x)$:

  - the expected value of X is given by

$$\overline{X} = \int_a^b x\ f(x)\ dx,$$

  - the standard deviation of X is given by

$$\sigma(X) = \sqrt{\mathrm{Var}(X)}$$

    where

$$\mathrm{Var}(X) = \int_a^b x^2\ f(x)\ dx - \left(\overline{X}\right)^2.$$