THE 7 CARDINAL SINS OF MATHEMATICS
(i.e. things you should never, ever do)

The following is a list of errors students routinely make when working on math problems. Committing any one of these errors in an assignment (be it a homework, quiz, or exam problem) will result in a huge penalty! In other words, make sure you never make these mistakes!

1.) **EXPONENTS ARE NOT ADDITIVE:** In other words,

\[(x + y)^p \neq x^p + y^p,\]

(unless \(p = 1\)). In particular \((x + y)^2 \neq x^2 + y^2\) and \(\sqrt{x + y} \neq \sqrt{x} + \sqrt{y}\) (except, of course, when either \(x\) or \(y\) is 0).

2.) **DENOMINATORS ARE NOT ADDITIVE:**

\[\frac{a}{b + c} \neq \frac{a}{b} + \frac{a}{c}.\]

Note that numerators are additive:

\[\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}.\]

3.) **CANCELLATION ERRORS:**

\[\frac{ab + c}{b} \neq a + c.\]

Another common type of cancellation error is

\[ax^n = x \rightarrow ax^{n-1} = 1.\]

Simply cancelling an \(x\) from both sides ignores the possibility that \(x = 0\) (and you cannot divide by 0). If you want to cancel a quantity involving a variable from both sides of an equation, you must always consider the case when the quantity is zero!

4.) **ZERO PRODUCT PROPERTY ERRORS:** The Zero Product Property (ZPP) says that if \(ab = 0\), then either \(a = 0\) or \(b = 0\). This is not the case if the right-hand side is not zero! In other words, if \(c \neq 0\) then

\[ab = c \text{ does not imply } a = c \text{ or } b = c.\]

As an example, if \(xy = 1\), we could have \(x = 2\) and \(y = 1/2\) (neither of which is equal to 1)!
5.) **TRIGONOMETRIC FUNCTIONS AND LOGARITHMS ARE NEITHER ADDITIVE NOR MULTIPLICATIVE:** That is,

\[
\begin{align*}
\sin(x + y) & \neq \sin(x) + \sin(y), \\
\sin(xy) & \neq \sin(x) \sin(y), \\
\ln(x + y) & \neq \ln(x) + \ln(y), \\
\ln(xy) & \neq \ln(x) \ln(y),
\end{align*}
\]

and similarly for the other trig functions (and logarithmic functions of other bases). Of course, these functions have identities which allow you to handle some of the left-hand sides above, but none are this simple.

6.) **DIFFERENTIATION IS NOT MULTIPLICATIVE:** Which means,

\[
(uv)' \neq u'v'.
\]

Instead, we have the product rule:

\[
(uv)' = u'v + uv'.
\]

7.) **INTEGRATION IS NOT MULTIPLICATIVE:** That is,

\[
\int f(x)g(x) \, dx \neq \left( \int f(x) \, dx \right) \left( \int g(x) \, dx \right).
\]

Instead, we have integration by parts:

\[
\int u \, dv = uv - \int v \, du.
\]