Computer Project 3
Python Implementation of Numeric ODE Solvers

DUE: Wednesday, November 9, 2022

1) Implement Euler's Method in Python. You should create a function called `Eulers_Method` declared as follows.

```
def Eulers_Method(func, x0, y0, xf, N):
```

The function arguments are:

- `func`: a function of the form `func(x,y)` which is the right-hand side of the standard first-order ODE

$$\frac{dy}{dx} = f(x, y),$$

- `x0, y0`: the initial condition for the ODE $y(x_0) = y_0$,
- `xf`: the final x–value that the method should predict a value for,
- `N`: the total number of steps used to get from `x0` to `xf`.

Your function should return two lists, `X` and `Y`. `X` is the list of `N+1` evenly spaced x-values starting from `x0` and ending at `xf`. `Y` should be the corresponding list of predicted y-values from Euler's Method.

2) Implement the Fourth Order Runge–Kutta Method in Python. You should create a function called `RK4` declared as follows.

```
def RK4(func, x0, y0, xf, N):
```

The function arguments and return types are exactly the same as for Euler's Method.

3) Find the exact solution to the following initial value problem by hand.

$$\frac{dy}{dx} + \frac{1}{x}y = \sqrt{y}$$
$$y(1) = 1$$

Use this to determine the value of $y(16)$.

4) Use your implementation of `Eulers_Method` from 1) to approximate the solution to the ODE from 3) on the interval $[1, 16]$ in 100 steps. Compute the absolute error between the exact value $y(16)$ and the final value reported from your method (which should be `Y[100]`).

5) Use your implementation of `Eulers_Method` from 1) to approximate the solution to the ODE from 3) on the interval $[1, 16]$ in 1000 steps. Compute the absolute error between the exact value $y(16)$ and the final value reported from your method (which should be `Y[1000]`). By what factor did the error improve from the run with 100 steps?

6) Use your implementation of `RK4` from 2) to approximate the solution to the ODE from 3) on the interval $[1, 16]$ in 100 steps. Compute the absolute error between the exact value $y(16)$ and the final value reported from your method (which should be `Y[100]`).

7) Use your implementation of `RK4` from 2) to approximate the solution to the ODE from 3) on the interval $[1, 16]$ in 1000 steps. Compute the absolute error between the exact value $y(16)$ and the final value reported from your method (which should be `Y[1000]`). By what factor did the error improve from the run with 100 steps?