# Introduction to Ordinary Differential Equations
# Academic Year 2022 – 2023

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REQUIRED TEXTS: *Differential Equations & Linear Algebra, 4th ed.*
Edwards, Penney, and Calvis
Pearson,
ISBN: 978-0-13-449718-1.

CLASS MEETINGS: MW 7:00PM – 8:30PM in Sprague 200

COURSE DESCRIPTION: First-order and linear higher order differential equations; matrices, determinants, and systems of differential equations; numerical and power series methods of solution; the Laplace transform.

DETAILED OBJECTIVES: Upon successful completion of the course, the student will have achieved the following objectives:

- Solve various types of first order equations, including separable, linear, exact, and Bernoulli equations, and be able to compute integrating factors and use them to solve differential equations and initial value problems.
- Solve second order linear equations, including the solution of inhomogeneous equations with the methods of undetermined coefficients and variation of parameters, and apply them in various settings, including vibrating springs and circuit problems.
- Use infinite series to solve nonlinear equations at ordinary points.
- Compute Laplace transforms and inverse Laplace transforms and use them to solve problems involving step functions and impulse functions.
- Solve systems of differential equations and apply them to problems like the mixing of solutions and coupled oscillators.
- Use matrix methods to solve systems of differential equations and higher-order equations.

- Solve differential equations numerically, using Euler's method, the modified Euler's method, and Runge-Kutta methods.
- Understand the theory of differential equations and differential operators, including existence and uniqueness results.

**CALCULATORS: THE USE OF CALCULATORS AND/OR CELL PHONES DURING EXAMS IS EXPRESSLY FORBIDDEN AND WILL RESULT IN A GRADE OF 0.**

GRADING: Five midterm exams and a cumulative final exam will be given. In addition, homework, quizzes, and computer projects will be given regularly. Your raw score will be computed as follows:

| **ITEM** | **% of Final Grade** |
|---|---|
| Homework | 10 % |
| Quizzes | 10 % |
| Computer Projects | 10 % |
| 5 Midterm Exams | 10 % each |
| Final Exam | 20% |
| TOTAL | 100 % |

Your final grade will be determined from your raw score as follows:

| **Raw Score** | 0 to 59.9 % | 60 to 64.9% | 65 to 69.9% | 70 to 74.9 % | 75 to 79.9 % | 80 to 84.9 % | 85 to 89.9 % | 90 to 100 % |
|---|---|---|---|---|---|---|---|---|
| **Grade** | 0 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |

MAKE-UP EXAMS: Make-up examinations will not be given except in the event of a verifiable emergency. If an extreme circumstance occurs which results in a student missing an exam, it is the student's responsibility to contact the instructor. Notification of the emergency should be made in a timely fashion, and proper documentation will be required.

ACADEMIC HONESTY: The work you hand in should be your own. If you receive help from any source, that help must be acknowledged in writing and turned in with the assignment. If there is evidence that work you hand in is not your own, the issue will be reported to the Honor Council Chair pursuant to the guidelines in the Student Handbook.

ROUGH COURSE OUTLINE

N. **Review and Preliminary Topics**

A. Review of Complex Numbers
B. Review of Systems of Linear Equations
C. Introduction to *Mathematica*
D. Introduction to Partial Derivatives

I. **Introduction to Ordinary Differential Equations**

A. Introduction to First Order Differential Equations
B. Analytic Solutions to Differential Equations
    1. Separable ODEs
    2. First Order Linear ODEs
    3. Exact ODEs
    4. Bernoulli and Homogeneous ODEs
C. Applications
D. Existence and Uniqueness Theorems
E. Numerical Methods
    1. Euler's Method
    2. Heun's Method
    3. Fourth Order Runge-Kutta
F. Autonomous ODEs and Bifurcations

II. **Linear Algebra**

A. Matrices as Transformations
B. Inverse Matrices and Determinants
C. Vector Spaces
D. Linear Independence and Bases
E. Linear Transformations
F. Eigenvalues and Eigenvectors
G. Diagonalization
H. The Matrix Exponential

III. **Higher Order Linear Differential Equations and Systems of Differential Equations**

A. General Theory of Linear ODEs
B. Linear Homogeneous ODEs with Constant Coefficients
C. Linear Inhomogeneous ODEs with Constant Coefficents
    1. Undetermined Coefficients
    2. Variation of Parameters

D. Applications
    1. Harmonic Oscillators: Springs and RLC Circuits
    2. Undamped Oscillators and Resonance
    3. Damped Oscillators

E. General Theory of Linear Systems of ODEs

F. Solutions to Homogeneous Linear Systems
    1. Diagonalizable Systems
    2. Non-Diagonalizable Systems

F. Inhomogeneous Linear Systems

H. Applications

IV. **Other Topics**

A. The Laplace Transform
    1. Definition and Basic Transforms
    2. Solving ODEs by Laplace Transform
    3. Step Functions, the Dirac Delta, and Convolutions

B. Power Series Techniques
    1. Series Solutions centered at Ordinary Points
    2. Series Solutions centered at Regular Singular Points

C. Non-Linear Autonomous Systems
    A. Equilibria
    B. Linearization and Classification for $2 \times 2$ Systems
    C. The Trace–Determinant Plane
    D. Bifurcation