Multivariable Calulus
Academic Year 2025 – 2026

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REQUIRED TEXTS: *Calculus, Early Transcendentals, 4th ed.,*
Rogawski and Adams,
W.H. Freeman and Co.,
ISBN: 978-1-4641-9377-4.

CLASS MEETINGS: MW 7:00PM – 8:30PM in Sprague 200

COURSE DESCRIPTION: Differential and integral calculus of real and vector valued functions. Topics include continuity, partial differentiation, implicit functions, Taylor's Theorem, gradient, curl, line, surface, and multiple integrals, inverse functions, theorems of Green and Stokes.

DETAILED OBJECTIVES: Upon successful completion of the course, the student will have achieved the following objectives:

- Understand lines and planes in 3-space.
- Work with polar, spherical and cylindrical coordinates.
- Classify and graph conic sections.
- Do vector computations in two and three dimensions, including dot products, cross products and triple products.
- Compute limits, derivatives and integrals of vector-valued functions, tangents and normals to curves and curvature.
- Compute partial derivatives, directional derivatives, gradients and find extreme values of functions of several variables.
- Use Lagrange Multipliers to find extreme values subject to constraints.
- Compute double and triple integrals in various coordinate systems.

- Work with parametrized surfaces.
- Work with vector fields and compute divergence and curl.
- Compute line integrals and surface integrals using Green's Theorem, Stokes' Theorem and the Divergence Theorem.

CALCULATORS: THE USE OF CALCULATORS, CELL PHONES, OR ANY OTHER ELECTRONIC DEVICES DURING EXAMS IS EXPRESSLY FORBIDDEN AND WILL RESULT IN A GRADE OF 0.

GRADING: Five midterm exams and a cumulative final exam will be given. In addition, homework and computer projects will be given regularly. Your raw score will be computed as follows:

ITEM	**Max. Pts.**
Quizzes	10 %
Computer Projects	5 %
Highest Two Midterm Exams Grade	16 % each
Middle Midterm Exam Grades	12%
Lowest Two Midterm Exam Grade	8 % each
Final Exam	25%
TOTAL	100 %

Your final grade will be determined from your raw score as follows:

Raw Score	0 to 59.9 %	60 to 64.9%	65 to 69.9%	70 to 74.9 %	75 to 79.9 %	80 to 84.9 %	85 to 89.9 %	90 to 100 %
Grade	0	1.0	1.5	2.0	2.5	3.0	3.5	4.0

MAKE-UP EXAMS: Make-up examinations will not be given except in the event of a verifiable emergency. If an extreme circumstance occurs which results in a student missing an exam, it is the student's responsibility to contact the instructor. Notification of the emergency should be made in a timely fashion, and proper documentation will be required.

ACADEMIC HONESTY: The work you hand in should be your own. If you receive help from any source, that help must be acknowledged in writing and turned in with the assignment. If there is evidence that work you hand in is not your own, the issue will be reported to the Honor Council Chair pursuant to the guidelines in the Student Handbook.

ROUGH COURSE OUTLINE

Unit I: **Geometry of $\mathbb{R}^2$ and $\mathbb{R}^3$: Vectors, Lines, Planes, and Matrices**

A. Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$
   1. Basics of Vectors and Applications
   2. Various Notations for Vectors
   2. Dot and Cross Products and their Geometric Interpretations

B. Lines in $\mathbb{R}^3$

C. Planes in $\mathbb{R}^3$

D. Linear Transformations of Vectors
   1. Linear Transformations of Vectors
   2. Matrix Operations
   3. Inverse Matrices and Determinants
   4. Eigenvalues and Eigenvectors
   5. Results for Real Symmetric Matrices

E. Special Surfaces in $\mathbb{R}^3$

Unit II: **Plane Curves and Space Curves**

A. Vector–Valued Functions of One Variable

B. Limits, Derivatives, and Integrals of Vector–Valued Functions

C. Projectile Motion

D. Speed and Arc Length

E. The TNB-Frame, Curvature, and Torsion

F. Motion in Space, Kepler's Laws

Unit III: **Differentiation of Functions of Several Variables**

A. Functions of Several Variables

B. Limits and Continuity

C. Partial Derivatives

D. Differentiability and Tangent Planes

E. The Gradient
   1. Directional Derivatives and the Gradient
   2. Geometric Interpretation of the Gradient

F. The Chain Rule

G. Optimization in Several Variables
   1. Critical Points
   2. The Second Derivative as a Matrix
   3. Second Derivative Test via Eigenvalues

H. Lagrange Multipliers

I. Taylor Series for Functions of Several Variables

Unit IV: **Multiple Integrals**

A. Integration in Two Variables (Rectangular Domains)
B. Double Integrals over More General Regions
C. Polar Coordinates and Double Integrals
D. Triple Integrals
E. Cylindrical and Spherical Coordinates
F. Change of Variables
G. Applications

Unit V: **Line and Surface Integrals**

A. Vector Fields
B. Line Integrals
    1. Line Integrals of Scalar Functions
    2. Line Integrals of Vector Fields
    3. Applications
C. Conservative Vector Fields, Path Independence, and the Fundamental Theorem of Line Integrals
D. Parametrized Surfaces
E. Surface Integrals
    1. Surface Integrals of Scalar Functions
    2. Surface Integrals of Vector Fields
    3. Applications

Unit VI: **Green, Stokes, and Gauss**

A. Green's Theorem
B. The Divergence and Curl
C. Stokes' Theorem and the Interpretation of the Curl
D. Gauss' Theorem (aka the Divergence Theorem) and the Interpretation of the Divergence
E. Applications to Fluid Mechanics and Electrodynamics