Intro. to ODEs

Quiz 15 **Solutions**

1) Compute the Laplace Transform of the following functions.

a) $f(t) = 3t^2 - 2t + 7$

$$F(s) = \frac{6}{s^3} - \frac{2}{s^2} + \frac{7}{s}$$

b) $g(t) = e^{-3t}\cos(4t)$

$$G(s) = \frac{s+3}{(s+3)^2+16} = \frac{s+3}{s^2+6s+25}$$

c) $h(t) = u(t-2)e^{9t}$

$$H(s) = e^{-2s}\mathcal{L}\left\{e^{9(t+2)}\right\}(s) = e^{-2s+18}\mathcal{L}\left\{e^{9t}\right\}(s) = \frac{e^{-2(s-9)}}{s-9}$$

2) Suppose that the Laplace Transform of a function f is

$$F(s) = \frac{s}{s^4+3s^2+1}.$$

Compute the Laplace Transform of $tf(t)$.

$$\begin{aligned}\mathcal{L}\{tf(t)\}(s) &= -\frac{d}{ds}[F(s)] \\ &= -\frac{d}{ds}\left[\frac{s}{s^4+3s^2+1}\right] \\ &= \frac{3s^4+3s^2-1}{(s^4+3s^2+1)^2}\end{aligned}$$

TURN OVER

3) Solve the following initial value problem by finding the Laplace Transform of the solution and then taking the inverse transform.

$$\begin{aligned} x'' + 4x &= 1 \\ x(0) &= 0 \\ x'(0) &= 2 \end{aligned}$$

$$\begin{aligned} s^2X - 2 + 4X &= \frac{1}{s} \\ (s^2+4)X &= \frac{1}{s} + 2 \\ X(s) &= \frac{1}{s(s^2+4)} + \frac{2}{s^2+4} \\ &= \frac{1}{4}\frac{1}{s} - \frac{1}{4}\frac{s}{s^2+4} + \frac{2}{s^2+4} \\ x(t) &= \frac{1}{4} - \frac{1}{4}\cos(2t) + \sin(2t) \end{aligned}$$