Intro. to ODEs

Quiz 13 **Solutions**

Find and classify all equilibria for the 2×2 non-linear system below.

$$\frac{dx}{dt} = (y-1)(y-x-2)$$
$$\frac{dy}{dt} = y(4-y-x)$$

For equilibria, we have

$$(y-1)(y-x-2) = 0 \longrightarrow y = 1, y = x+2$$
$$y(4-y-x) = 0 \longrightarrow y = 0, y = 4-x$$

which gives $(x, y) = (3, 1), (-2, 0)$, and $(1, 3)$.

The matrix of partial derivatives we need to consider is

$$L(x, y) = \begin{bmatrix} -(y-1) & 2y-x-3 \\ -4 & 4-x-2y \end{bmatrix}$$

which has a trace of $\tau(x, y) = 5 - x - 3y$ and determinant of

$$\Delta(x, y) = 4y^2 - 9y - x + 4.$$

(These functions are *only* significant at the equilibria.)

- $(3, 1)$: $\tau = -1, \Delta = -4$

  So, $(3, 1)$ is an *unstable saddle point.*

- $(-2, 0)$: $\tau = 7, \Delta = 6, \Delta < \frac{1}{4}\tau^2$

  So, $(-2, 0)$ is an *unstable node.*

- $(1, 3)$: $\tau = -5, \Delta = 12, \Delta > \frac{1}{4}\tau^2$

  So, $(0, 0)$ is an *asymptotically stable spiral point.*