Part A: Problem 1

We begin by defining the given function and finding its real zeros. Notice that the Solve returns replacement rules which we will say more about later.

In[2]:=
```
f[x_] := x^4 + 2 x^3 - 17 x^2 - 4 x + 30
Solve[f[x] == 0, x, Reals]
```

Out[3]= $\{\{x \to -5\}, \{x \to 3\}, \{x \to -\sqrt{2}\}, \{x \to \sqrt{2}\}\}$

Next, we find the first two derivatives and their zeros. Notice the use of NSolve to get decimal approximations to the critical points and inflection points (the zeros of the second derivative are actually not terribly ugly).

In[8]:=
```
NSolve[D[f[x], x] == 0, x, Reals]
NSolve[D[f[x], {x, 2}] == 0, x, Reals]
```

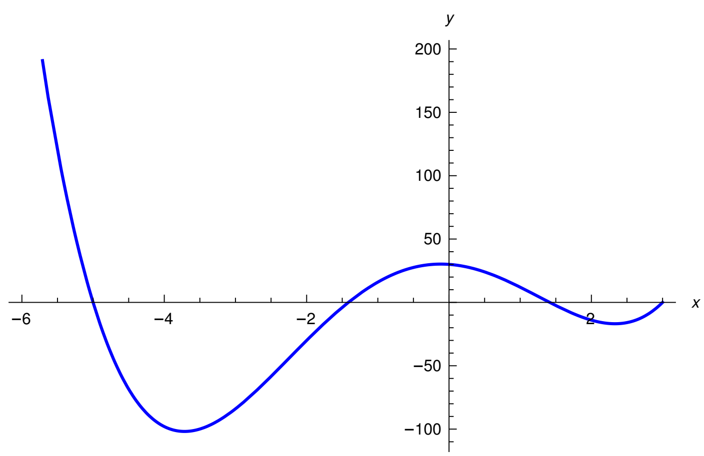
Out[8]= {{x → -3.71536}, {x → -0.115475}, {x → 2.33083}}

Out[9]= {{x → -2.25594}, {x → 1.25594}}

Finally, we give a plot of the function on [-6, 3].

In[10]:=
```
Plot[f[x], {x, -6, 3}, AxesLabel → {x, y}, PlotStyle → Blue]
```

Out[10]=



Part A: Problem 2

This problem goes exactly like the first.

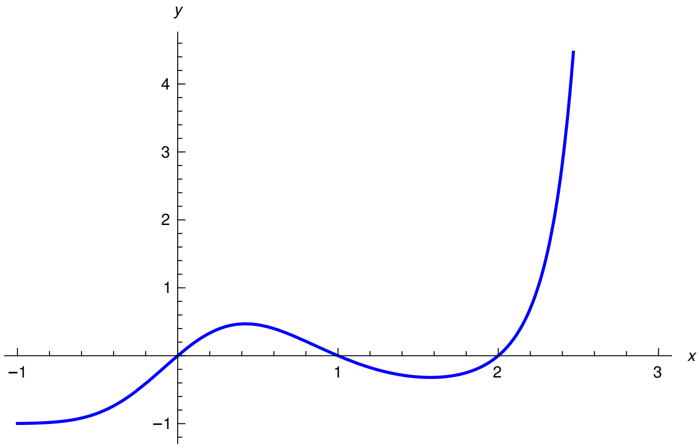
In[11]:=
```
g[x_] := Exp[x^3 - 3 x^2 + 2 x] - 1
Solve[g[x] == 0, x, Reals]
NSolve[D[g[x], x] == 0, x, Reals]
NSolve[D[g[x], {x, 2}] == 0, x, Reals]
```

Out[12]= {{x → 0}, {x → 1}, {x → 2}}

Out[13]= {{x → 0.42265}, {x → 1.57735}}

Out[14]= {{x → -0.0887026}, {x → 0.853975}}

In[15]:=
```
Plot[g[x], {x, -1, 3}, AxesLabel → {x, y}, PlotStyle → Blue]
```

Out[15]=


Part B: Problem 1

We begin by finding the exact intersection of the two curves. We also get numerical values for plotting.

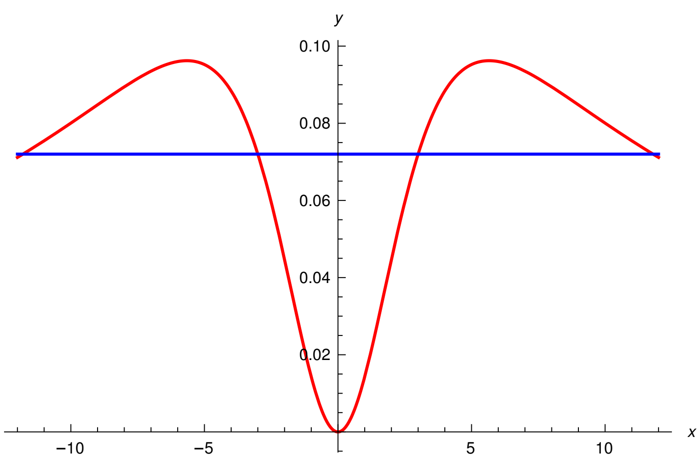
In[28]:=
```
Solve[x^2 / (x^2 + 16)^(3 / 2) == 9 / 125, x, Reals]
NSolve[x^2 / (x^2 + 16)^(3 / 2) == 9 / 125, x, Reals]
```

Out[28]= $\left\{\{x \to -3\}, \{x \to 3\}, \left\{x \to -\frac{8}{9}\sqrt{86 + 25\sqrt{13}}\right\}, \left\{x \to \frac{8}{9}\sqrt{86 + 25\sqrt{13}}\right\}\right\}$

Out[29]= {{x → −11.7971}, {x → −3.}, {x → 3.}, {x → 11.7971}}

Notice in the option “PlotRange -> Full” in the command below. This forces Mathematica to display the entire graph.

In[27]:=
```
Plot[{x^2 / (x^2 + 16)^(3 / 2), 9 / 125}, {x, -12, 12},
 AxesLabel → {x, y}, PlotStyle → {Red, Blue}, PlotRange → Full]
```

Out[27]=


Given the symmetry, we can simply find the area of the right half and multiply by two. Notice the use of % in the second command. This instructs Mathematica to replace the % sign with the previous output.

```
In[32]:= 2 * (Integrate[9 / 125 - x ^ 2 / (x ^ 2 + 16) ^ (3 / 2), {x, 0, 3}] +
    Integrate[x ^ 2 / (x ^ 2 + 16) ^ (3 / 2) - 9 / 125, {x, 3, 8/9 Sqrt[86 + 25 Sqrt[13]]}])
N[
 %,
 10]
```

Out[32]= $2\left(\frac{102}{125}-\frac{2}{125}\left(-51+\sqrt{6026+1825\sqrt{13}}\right)-\text{ArcSinh}\left[\frac{3}{4}\right]+\text{ArcSinh}\left[\frac{2}{9}\sqrt{86+25\sqrt{13}}\right]-\text{Log}[2]\right)$

Out[33]= 0.5031102632

Part B: Problem 2

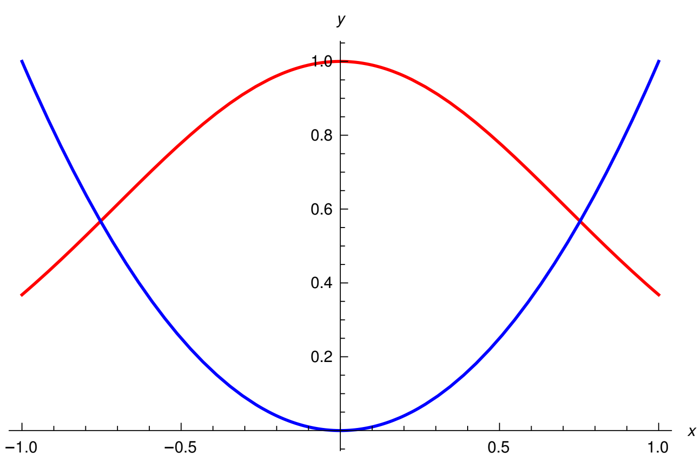
```
In[38]:= Solve[Exp[-x ^ 2] == x ^ 2, x, Reals]
NSolve[Exp[-x ^ 2] == x ^ 2, x, Reals]
```

Out[38]= $\left\{\left\{x \to -\sqrt{\text{ProductLog}[1]}\right\}, \left\{x \to \sqrt{\text{ProductLog}[1]}\right\}\right\}$

Out[39]= {{x → -0.753089}, {x → 0.753089}}

```
In[37]:= Plot[{Exp[-x ^ 2], x ^ 2}, {x, -1, 1},
  AxesLabel → {x, y}, PlotStyle → {Red, Blue}, PlotRange → Full]
```

Out[37]=



In[40]:=
```
2 * Integrate[Exp[-x^2] - x^2, {x, 0, Sqrt[ProductLog[1]]}]
N[%, 10]
```

Out[40]= $\frac{1}{3}\left(3\sqrt{\pi}\ \text{Erf}\left[\sqrt{\text{ProductLog}[1]}\right]-2\ \text{ProductLog}[1]^{3/2}\right)$

Out[41]= 0.9792630484