Mathematica Project 5 Help

Part 1: Surfaces of the Form z = f(x, y)

For a surface of the form z = f(x,y) at a point P = (a,b), the tangent plane is given by $z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$. In order to compute the partial derivates efficiently, we use the Grad[...] command to compute the gradient. We then pass it the point P = (1,2) by giving those values as a replacement rule outside of the Grad command.

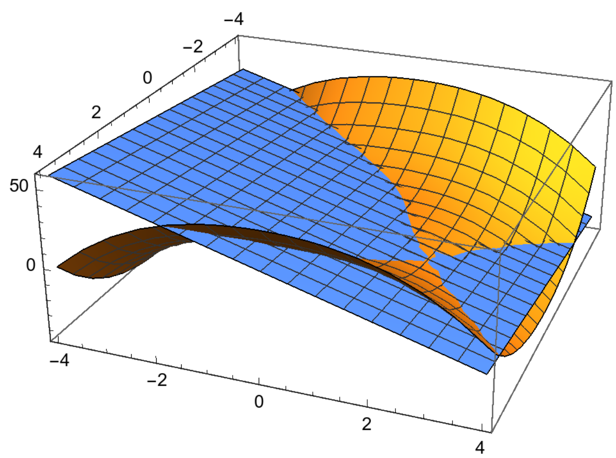
```
In[3]:= f[x_, y_] := (x^2 - y^2) * Log[1 + x^2 + y^2]
        Grad[f[x, y], {x, y}] /. {x → 1, y → 2}

Out[4]= {-1 + 2 Log[6], -2 - 4 Log[6]}

In[5]:= TP[x_, y_] := f[1, 2] + (2 Log[6] - 1) * (x - 1) + (-4 Log[6] - 2) * (y - 2)

In[7]:= Plot3D[{f[x, y], TP[x, y]}, {x, -4, 4}, {y, -4, 4}]
```

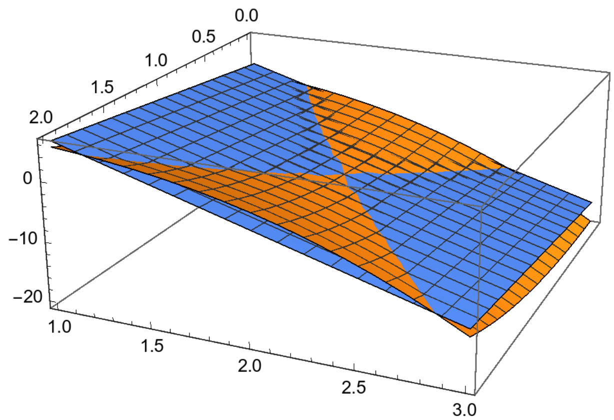
Out[7]=



To demonstrate that the tangent plane becomes a better approximation as we approach the base point, we give a second graph that is more zoomed in on the point P = (1,2).

In[8]:=
```
Plot3D[{f[x, y], TP[x, y]}, {x, 0, 2}, {y, 1, 3}]
```

Out[8]=



Part 2: Surfaces of the Form F(x, y, z) = c

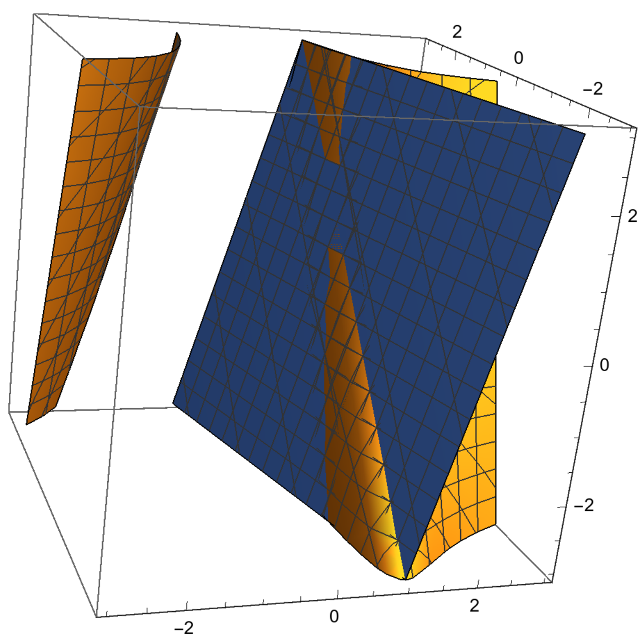
For the level surface of a function of three variables (i.e. a surface of the form F(x, y, z) = c), the tangent plane at a given point P = (a, b, c) can be written in the form ∇f(a, b, c) • ⟨x - a, y - b, z - c⟩ = 0. Below, notice that the surface $x^2\, y - x\, z + 3\, x = 3$ goes through the point Q = (1, 1, 1).

In[9]:=
```
F[x_, y_, z_] := x^2 y - x * z + 3 x
Grad[F[x, y, z], {x, y, z}] /. {x → 1, y → 1, z → 1}
```

Out[10]= {4, 1, -1}

In[11]:=
```
ContourPlot3D[{F[x, y, z] == 3, 4 (x - 1) + 1 (y - 1) - 1 (z - 1) == 0},
 {x, -3, 3}, {y, -3, 3}, {z, -3, 3}]
```

Out[11]=



Again, we zoom in on the point Q = (1,1,1) to show that the tangent plane becomes a better approxima·

tion of the function near the base point.

In[12]:=
```
ContourPlot3D[{F[x, y, z] == 3, 4 (x - 1) + 1 (y - 1) - 1 (z - 1) == 0}, {x, 0, 2}, {y, 0, 2}, {z, 0, 2}]
```

Out[12]=