

## Mathematica Assignment 2

### Eigenvalues

**Instructions:** Create a *Mathematica* notebook that solves the following problems. Email your completed notebook to me at byoung@wyomingseminary.org with the Subject: Mathematica 2.

For the matrices in 1) and 2), use *Mathematica* to find the characteristic polynomial. Then use `NSolve` to find numerical approximations to the eigenvalues.

1)

$$M_1 = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -5 \\ 1 & -5 & 7 \end{bmatrix}$$

2)

$$M_2 = \begin{bmatrix} 1 & -2 & -1 & 7 \\ -2 & 0 & 5 & -2 \\ -1 & 5 & 6 & -4 \\ 7 & -2 & -4 & 10 \end{bmatrix}$$

3) The matrix

$$R_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

rotates 2-dimensional vectors counter-clockwise by the angle  $\theta$ .

- a) What is the determinant of this matrix?
- b) Find the eigenvalues of this matrix in terms of  $\theta$ .
- c) Over the range  $0 \leq \theta < 2\pi$ , for what values of  $\theta$  are the eigenvalues real?
- d) Why are the eigenvalues complex for most values of  $\theta$ ? (Hint: Think about the geometric interpretation of eigenvectors for real eigenvalues.)

## Some Helpful Commands

- Matrices in *Mathematica* are implemented as nested lists of rows. For example, the matrix  $M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$  is entered  
 $M = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}\}$ .
- `CharacteristicPolynomial[matrix, variable]` gives the characteristic polynomial of `matrix` using `variable` for the variable in the polynomial.
- `Solve[expression, variables]` finds exact solutions to the equation given in `expression` which is in terms of `variables`. If you give `Solve` an expression with a decimal, it defaults to `NSolve`. Remember to use the logical operator `==` in your equation rather than the assignment operator `=`.
- `NSolve[expression, variables]` numerically finds solutions to the equation given in `expression` which is in terms of `variables`. Remember to use the logical operator `==` in your equation rather than the assignment operator `=`.