COMPUTER PROJECT 9
Surface Area and Volume of a Torus

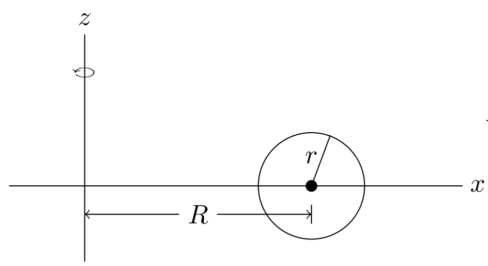
DUE: 05/09/2022

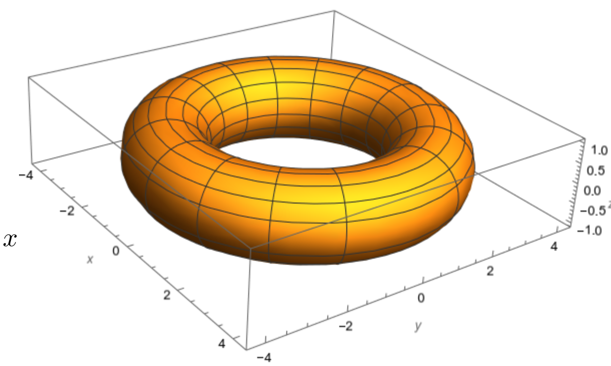
Instructions: Use *Mathematica* to solve the following problems. Email your notebook file to me at *byoung@wyomingseminary.org*, and use "Mathematica Project 9" as the subject line of your email.

The surface of a torus can be parametrized by

$$\vec{r}(\theta, \varphi) = \left\langle \Big(R + r\cos(\varphi)\Big)\cos(\theta),\ \Big(R + r\cos(\varphi)\Big)\cos(\theta),\ r\sin(\varphi) \right\rangle$$

where both angles are in the range $0 \le \theta \le 2\pi$ and $0 \le \varphi \le 2\pi$. Here, r is the smaller, inner radius of the torus while R is the larger outer radius (both are considered to be fixed numbers).





a) Show that the surface area of the torus is

$$\iint_{\text{torus}} 1\, dS = 4\pi^2 r R.$$

b) Use the divergence theorem to show that

$$\text{vol(torus)} = \frac{1}{3}\iint_{\text{torus}} \langle x,\ y,\ z\rangle \cdot d\vec{S} = (2\pi R)(\pi r^2).$$