Multivariable Calculus
Homework 3 Solutions

Section 8.9 pp. 432 – 436

2.) $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 16 \\ 1 & 3 & -1 & 22 \\ 2 & 5 & 0 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

So, this system of equations has no solutions.

4.) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -2 & 1 & 11 \\ 3 & 1 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$

So, this system of equations has a unique solution given by
$x = 2, y = -3$, and $z = 1$.

6.) $\left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 3 & 1 & 2 & -7 \\ 5 & 4 & 2 & -4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -4 \end{array}\right]$

So, this system of equations has a unique solution given by
$x = 0, y = 1$, and $z = -4$.

12.)

a) $2A = \begin{bmatrix} 4 & 4 & 6 \\ 2 & -6 & 8 \end{bmatrix}$

b) $-3C = \begin{bmatrix} 6 & 0 & -3 \\ -15 & -6 & 9 \\ 3 & 3 & 3 \end{bmatrix}$

c) $C + 2D$ does not exist since the two matrices are different sizes.

d) $2A - 3B = \begin{bmatrix} 1 & 7 & 0 \\ 2 & -21 & -13 \end{bmatrix}$

13.)

a) AB does not exist since since A has 3 columns but B has only 2 rows.

b) $AC = \begin{bmatrix} 3 & 1 & -7 \\ -21 & -10 & 6 \end{bmatrix}$

c) $DA = \begin{bmatrix} -2 & 14 & -13 \\ 1 & -19 & 14 \end{bmatrix}$

d) $D^2 = \begin{bmatrix} 9 & -24 \\ -12 & 33 \end{bmatrix}$

e) $C^2 = \begin{bmatrix} 3 & -1 & -3 \\ 3 & 7 & 2 \\ -2 & -1 & 3 \end{bmatrix}$

14.)

a) $\begin{vmatrix} 1 & -3 \\ 2 & 5 \end{vmatrix} = 11$

b) $\begin{vmatrix} 1 & -2 & -3 \\ 4 & -1 & 0 \\ -3 & -1 & 1 \end{vmatrix} = 28$

c) $\begin{vmatrix} 1 & 2 & -3 \\ 2 & 1 & 1 \\ 1 & -1 & 4 \end{vmatrix} = 0$

d) $\begin{vmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 2 \\ 1 & -3 & 2 & -1 \end{vmatrix} = -6$

15.)

a) $\left[\begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ -1 & 6 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{3}{8} & -\frac{1}{4} \\ 0 & 1 & \frac{1}{16} & \frac{1}{8} \end{array}\right]$

So, the inverse matrix is

$$\frac{1}{16}\begin{bmatrix} 6 & -4 \\ 1 & 2 \end{bmatrix}.$$

b) $\left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & 0 & 2/3 \\ 0 & 1 & 0 & 2/3 & -1 & 4/3 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right]$

So, the inverse matrix is

$$\frac{1}{3}\begin{bmatrix} 1 & 0 & 2 \\ 2 & -3 & 4 \\ 3 & -3 & 3 \end{bmatrix}.$$

c) $\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 4 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & 2/3 & 1/3 \\ 0 & 1 & 0 & 4/3 & -5/3 & -1/3 \\ 0 & 0 & 1 & -4/3 & 8/3 & 1/3 \end{array}\right]$

So, the inverse matrix is

$$\frac{1}{3}\begin{bmatrix} -1 & 2 & 1 \\ 4 & -5 & -1 \\ -4 & 8 & 1 \end{bmatrix}.$$

d) $\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 5 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 2 & 0 & 1/2 & -7/2 & 1/2 \\ 0 & 1 & 0 & -1 & 0 & 1/2 & 3/2 & -1/2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 5/2 & -1/2 \end{array}\right]$

Since we failed to row reduce the left half to the identity matrix, the original matrix is not invertible and has determinant zero.

16.)

a)

$$M = \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix} \longrightarrow M^{-1} = -\frac{1}{7}\begin{bmatrix} -3 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 11 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

So, $x = 2$ and $y = -3$.

b)

$$M = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ -1 & 5 & -2 \end{bmatrix} \longrightarrow M^{-1} = \frac{1}{8}\begin{bmatrix} -11 & 9 & -1 \\ 3 & -1 & 1 \\ 13 & -7 & -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -11 & 9 & -1 \\ 3 & -1 & 1 \\ 13 & -7 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

So, $x = -2, y = 1$ and $z = 3$.