Exam 5 Review Problems

1. Find the centroid of half of a right circular cone of radius R and height H. You can take this to be the cone

$$z = \frac{H}{R}\sqrt{x^2 + y^2}$$

bounded above by the plane $z = H$ and with $x \geq 0$.

2. Find the center of mass of the region between the paraboloid $z = r^2$ and the xy-plane ($z = 0$) and inside the cylinder of $r = 1$ if the density of the region is $\delta(r, \theta, z) = z$.

3. Compute the line integral $\int_{\mathcal{C}} f\, dr$ for the scalar function

$$f(x, y, z) = \frac{\sqrt{3}}{x^2 + y^2 + z^2}$$

over the curve $\mathcal{C}$ which is the semi-infinite ray from the origin in the direction $\langle 1, 1, 1\rangle$.

4. Compute the line integral $\int_{\mathcal{C}} \vec{f}\cdot d\vec{r}$ for the vector field $\vec{f}(x, y, z) = -y\hat{\imath} + x\hat{\jmath} + 2\hat{k}$ along the helix $\mathcal{C}$ given by $\vec{r}(t) = \langle -2\cos(t),\ 2\sin(t),\ 2t\rangle$ with $0 \leq t \leq 2\pi$.

5. Compute

$$\int_{\mathcal{C}} \left\langle \frac{2x}{y},\ \frac{1 - x^2}{y^2} \right\rangle \cdot\ d\vec{r}$$

where $\mathcal{C}$ is the semi-circular arc starting at $(-2, 1)$ passing through $(0, 3)$ and ending at $(2, 1)$.

6. Verify that the vector field

$$\vec{F}(x, y, z) = \left\langle \ln(x) + \sec^2(x + y),\ \sec^2(x + y) + \frac{y}{y^2 + z^2},\ \frac{z}{y^2 + z^2} \right\rangle$$

is conservative over the first octant ($x, y, z > 0$). Find a potential function $\varphi(x, y, z)$ for this vector field over that region.

7. Compute the line integral of the vector field

$$\vec{F}(x, y) = \left(\frac{3}{2}y^2 - x^2y + \arctan(2x^2 + 4x + 1)\right)\hat{\imath} + \left(\frac{1}{3}x^3 + 3xy + \sqrt{e^{y^2} + 2}\right)\hat{\jmath}$$

over the plane curve starting at the origin, moving to $(2, 0)$ along the x-axis, then moving vertically from $(2, 0)$ to $(2, 6)$, and finally moving along the curve $y = 5x - x^2$ from $(2, 6)$ back to the origin.

8. Compute the counter-clockwise circulation and outward flux for the vector field $\vec{f} = \langle x + e^x \sin(y),\ x + e^x \cos(y)\rangle$ over the right-hand loop of the lemniscate $r^2 = \cos(\theta)$.