Exam 5 Review Problems

1. Compute the line integral $\int_{\mathcal{C}} f\, ds$ for the scalar function

$$f(x,y,z) = \frac{\sqrt{3}}{3 + x^2 + y^2 + z^2}$$

over the curve $\mathcal{C}$ which is the semi-infinite ray from the origin in the direction $\langle 1,1,1\rangle$.

2. Compute the line integral $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F}(x,y,z) = -y\hat{\imath} + x\hat{\jmath} + 2\hat{k}$ along the helix $\mathcal{C}$ given by $\vec{r}(t) = \langle -2\cos(t),\ 2\sin(t),\ 2t\rangle$ with $0 \le t \le 2\pi$.

3. Compute

$$\int_{\mathcal{C}} \left\langle \frac{2x}{y},\ \frac{1-x^2}{y^2} \right\rangle \cdot\, d\vec{r}$$

where $\mathcal{C}$ is the semi-circular arc starting at $(-2,1)$ passing through $(0,3)$ and ending at $(2,1)$.

4. Verify that the vector field

$$\vec{F}(x,y,z) = \left\langle \ln(x) + \sec^2(x+y),\ \sec^2(x+y) + \frac{y}{y^2+z^2},\ \frac{z}{y^2+z^2} \right\rangle$$

is conservative over the first octant $(x,y,z > 0)$. Find a potential function $f(x,y,z)$ for this vector field over that region.

5. Compute

$$\int_{\mathcal{C}} \left(y^2\cos(z)\hat{\imath} + 2xy\cos(z)\hat{\imath} - xy^2\sin(z)\hat{k} \right) \cdot d\vec{r}$$

where the curve $\mathcal{C}$ is given by

$$\vec{r}(t) = \langle t^2,\ \sin(t),\ t\rangle \text{ over the range } 0 \le t \le 2\pi.$$

6. Find the surface area of the portion of the sphere $x^2+y^2+z^2 = 4$ lying above the plane $z = \sqrt{2}$.

7. Compute $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$ for the vector field

$$\vec{F}(x,y,z) = \langle xy, yz, xz\rangle$$

over the surface $\mathcal{S}$ which is the portion of the paraboloid $z = 4 - x^2 - y^2$ lying over the semicircle of radius 1 centered at the origin with $y \ge 0$ with orientation toward from the z-axis.

8. Compute $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$ for the vector field

$$\vec{F}(x,y,z) = \langle x, -z, y\rangle$$

over the surface $\mathcal{S}$ which is the portion of the cone $z = \sqrt{x^2+y^2}$ lying below the plane $z = 3$ with orientation away from the z-axis.