

# Exam 2 Outline

## Calculus of Vector-Valued Functions of a Single Variable: Plane Curves and Space Curves

- I. Vector-Valued Functions of One Variable
  - A. Be able to parametrize curves described geometrically.
  - B. Be able to recognize basic plane and space curves: lines, circles, ellipses, and helices.
- II. Limits, Derivatives, and Integrals of Vector-Valued Functions
  - A. Be able to compute limits, derivatives, and integrals of vector-valued functions of a single variable.
  - B. Understand the connection between plane and space curves and the trajectory of a particle; in particular, know how to compute velocity and acceleration of moving particles.
  - C. Know basic differentiation rules especially as they relate to the dot and cross products.
  - D. Be able to prove that if  $|\vec{r}(t)| = C$  (a constant), then  $\vec{r}'(t)$  is orthogonal to  $\vec{r}(t)$ .
- III. Projectile Motion
  - A. Be able to compute the motion of a point particle launched from the origin at a given velocity in a constant gravitational field.
  - B. Be able to answer basic questions about the motion (highest point above the ground, time of flight, total horizontal distance traveled).
- IV. Speed and Arc Length
  - A. Be able to compute the speed of a parametrized curve, and use it to compute arc lengths of sections of curves.
  - B. Be able to compute the arc length function for a given (simple) parametrized curve as measured from a specified point on the curve, and use it to find the arc length parametrization of the curve based at the given point.
- V. The *TNB*-Frame and Curvature
  - A. Be able to compute the unit tangent, unit normal, and unit binormal for a given space curve.
  - B. Be able to compute the curvature of a given space curve.
- VI. Motion in Space
  - A. Be able to compute the tangential and normal components of acceleration.
  - B. Know the acceleration of a particle moving on a circle at constant speed.

## Exam 2 Formula Sheet

- For a regular curve  $\vec{r}(t)$ ,

$$- \vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|}$$

$$- \vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

$$- \vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

- For a regular curve  $\vec{r}(t)$ , the curvature is given by

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}.$$