THE 7 CARDINAL SINS OF MATHEMATICS
(i.e. things you should never, ever do)

The following is a list of errors students routinely make when working on math problems. Committing any one of these errors in an assignment (be it a homework, quiz, or exam problem) will result in a *huge* penalty! In other words, make sure you **never make these mistakes**!

1.) **EXPONENTS ARE NOT ADDITIVE:** In other words,

$$(x+y)^p \neq x^p + y^p,$$

(unless $p = 1$). In particular $(x+y)^2 \neq x^2+y^2$ and $\sqrt{x+y} \neq \sqrt{x}+\sqrt{y}$ (except, of course, when either x or y is 0).

2.) **DENOMINATORS ARE NOT ADDITIVE:**

$$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}.$$

Note that **numerators are additive**:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}.$$

3.) **CANCELLATION ERRORS:**

$$\frac{ab+c}{b} \neq a + c.$$

Another common type of cancellation error is

$$ax^n = x \nrightarrow ax^{n-1} = 1.$$

Simply cancelling an x from both sides ignores the possibility that $x = 0$ (and you cannot divide by 0). If you want to cancel a quantity involving a variable from both sides of an equation, you must **always** consider the case when the quantity is zero!

4.) **ZERO PRODUCT PROPERTY ERRORS:** The Zero Product Property (ZPP) says that if $ab = 0$, then either $a = 0$ or $b = 0$. This is not the case if the right-hand side is not zero! In other words, if $c \neq 0$ then

$$ab = c \ \textbf{ does not imply } \ a = c \text{ or } b = c.$$

As an example, if $xy = 1$, we could have $x = 2$ and $y = 1/2$ (neither of which is equal to 1)!

5.) **TRIGONOMETRIC FUNCTIONS AND LOGARITHMS ARE NEITHER ADDITIVE NOR MULTIPLICATIVE:** That is,

$$\begin{aligned} \sin(x+y) &\neq \sin(x)+\sin(y), \\ \sin(xy) &\neq \sin(x)\sin(y), \\ \ln(x+y) &\neq \ln(x)+\ln(y), \\ \ln(xy) &\neq \ln(x)\ln(y), \end{aligned}$$

and similarly for the other trig functions (and logarithmic functions of other bases). Of course, these functions have identities which allow you to handle some of the left-hand sides above, but none are this simple.

6.) **DIFFERENTIATION IS NOT MULTIPLICATIVE:** Which means,

$$(uv)' \neq u'v'.$$

Instead, we have the product rule:

$$(uv)' = u'v + uv'.$$

7.) **INTEGRATION IS NOT MULTIPLICATIVE:** That is,

$$\int f(x)g(x)\,dx \neq \left(\int f(x)\,dx\right)\left(\int g(x)\,dx\right).$$

Instead, we have integration by parts:

$$\int u\,dv = uv - \int v\,du.$$