FORMULA SHEET

- For an equation of the form
$$\frac{dy}{dx} + P(x)y = Q(x),$$
the general solution is of the form
$$y(x) = \frac{\int \mu(x)Q(x)\ dx + C}{\mu(x)},$$
where
$$\mu(x) = e^{\int P(x)\ dx}.$$
- A first order homogeneous equation can be reduced to a separable differential equation by the substitution
$$y(x) = xv(x).$$
- A Bernoulli equation can be reduced to a first order linear differential equation by the substitution
$$z(x) = [y(x)]^{1-n}.$$
- A differential equation of the form
$$M(x,y)dx + N(x,y)dy = 0$$
is an *exact equation* if and only if
$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$
For an exact equation, the general solution is of the form $F(x,y) = C$ where
$$\frac{\partial F}{\partial x} = M \quad \text{and} \quad \frac{\partial F}{\partial y} = N.$$
- **Variation of Parameters:** Suppose $x_1(t)$ and $x_2(t)$ form a complete set of solutions to the homogenous equation
$$x'' + p(t)x' + q(t)x = 0.$$
That is, the general homogenous solution is of the form
$$x_h(t) = c_1x_1(t) + c_2x_2(t).$$
A particular solution to the inhomogeneous equation
$$x'' + p(t)x' + q(t)x = f(t)$$
is given by
$$x_p(t) = v_1(t)x_1(t) + v_2(t)x_2(t)$$
where
$$v_1(t) = -\int \frac{x_2(t)f(t)}{W[x_1,x_2](t)}dt$$
and
$$v_2(t) = \int \frac{x_1(t)f(t)}{W[x_1,x_2](t)}dt,$$
where the Wronskian is given by
$$W[x_1,x_2](t) = \begin{vmatrix} x_1(t) & x_2(t) \\ x_1'(t) & x_2'(t) \end{vmatrix}.$$