

Introduction to Ordinary Differential Equations

Outline for Exam 4

Test Date: 02/22/2023

NO BOOKS OR NOTES WILL BE PERMITTED! NO ELECTRONIC DEVICES ARE PERMITTED!

I. Linear Systems of Differential Equations

A. Basics of First Order Systems

1. Be able to rewrite a linear system in matrix/vector form.
2. Be able to rewrite higher order ODEs as linear systems of first order ODEs.

B. Solutions to First Order Systems by Eigenvalues/Eigenvectors

1. Be able to write the general solution to a diagonalizable linear system with constant coefficients by computing eigenvalues and eigenvectors.
2. In the case of complex eigenvalues, be able to write the solution in real form.
3. Be able to handle repeated eigenvalues with generalized eigenvectors in simple cases (2×2 and 3×3 matrices only).
4. Be able to find particular solutions for simple inhomogeneous terms using Undetermined Coefficients.

II. Applications of First Order Systems

- A. Be able to setup and solve problems involving coupled tanks.
- B. Be able to setup and solve problems involving coupled masses and springs.

III. The Matrix Exponential

- A. Be able to compute a fundamental matrix solution $\Phi(t)$ for a linear system of the form

$$\frac{d\vec{x}}{dt} = M\vec{x}.$$

- B. For small matrices (i.e. 2×2), be able to compute e^{tM} using a fundamental matrix.

$$e^{tM} = \Phi(t)\Phi^{-1}(0)$$